HYPERBOLIC 3-MANIFOLDS ADMITTING NO FILLABLE CONTACT STRUCTURES

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Abstract. In this paper, we find infinite hyperbolic 3-manifolds that admit no weakly symplectically fillable contact structures, using tools in Heegaard Floer theory. We also remark that part of these manifolds do admit tight contact structures.

1. Introduction

There is a dichotomy for the contact structures on oriented 3-manifolds introduced by Eliashberg: tight and overtwisted contact structures. The complete classification of overtwisted contact structures has been accomplished by Eliashberg in [El1]. However, the fundamental questions on the existence and classification of tight contact structures have still not been completely solved, and they are widely open especially for hyperbolic 3-manifolds. According to [El2], a weakly symplectically fillable contact structure is known to be tight.

Question 1.1 (Question 8.2.2, [El3] and Problem 4.142, [Ki])**.** Given an irreducible 3-manifold M, does it admit a tight or fillable contact structure?

There has not been a general method to answer Question 1.1 for an arbitrary 3-manifold M. However, many important examples have been studied. The first example of a 3-manifold that admits no weakly symplectically fillable contact structures has been found in [Li1, Corollary 1.5]. It is the Poincaré homology sphere with its natural orientation reversed. In fact, it does not admit any tight contact structures either. See [EtH1]. Subsequently, more examples of 3-manifolds that admit no weakly symplectically fillable contact structures have been constructed. See [Li2, Theorem 2.1], [LiSt2, Theorem 4.2], [LiSt3, Proposition 4.1] and [OwSt2, Corollary 3]. All these examples are either Seifert fibered spaces or reducible 3-manifolds. All of such Seifert fibered spaces are eventually classified in [LeLi, Theorem 1.5]. Furthermore, the existence problem of tight contact structures on Seifert fibered spaces has been solved in [LiSt4]. Thus, Question 1.1 has been completely solved for all Seifert fibered spaces.

However, less examples of hyperbolic 3-manifolds have been studied. Basically, the techniques used in Seifert fibered spaces are hard to apply to hyperbolic manifolds. Until the present paper, it wasn't known whether there is a hyperbolic 3-manifold which admits no fillable contact structures. Tools in Heegaard Floer homology enable us to find such examples. In this paper, we give infinitely many hyperbolic manifolds which do not admit any weakly fillable contact structures.

Theorem 1.2. *Suppose $q \geq 4$ is a positive integer. Let M_q be the $(2q+3)$-surgery on S^3 along the pretzel knot $P(-2,3,2q+1)$. If $2q+3$ is square free, then M_q is a hyperbolic 3-manifold that admits no weakly symplectically fillable contact structures.*

Since each prime number greater than 9 is equal to some $2q+3$ for some integer q, we can find infinitely many q such that $2q+3$ is square-free. Thus, we have obtained an infinite family of hyperbolic 3-manifolds which admit no weakly symplectically fillable contact structures. While usually such proofs resort to Donaldson's celebrated diagonalization theorem or Seiberg-Witten theory, our proof relies on a theorem of Owens-Strle on the correction terms and a theorem of Ozsváth-Szabó on L-spaces in Heegaard Floer theory.

The 3-manifold M_q, where $q \geq 4$ and $2q+3$ is square-free, may also serve as a potential example of hyperbolic 3-manifold which admits no tight contact structures. We raise the following question.

Question 1.3. Suppose $q \geq 4$, does M_q admit a tight contact structure?

By computer experiments, we can find more examples of surgeries on $P(-2,3,2q+1)$ admitting no fillable contact structures, including rational surgeries on knots. Here, we give a proof for the following sequence of rational surgeries on $P(-2,3,7)$.

Theorem 1.4. *Suppose $p \geq 1$ is an integer. Let M'_p be the $\frac{10p+1}{p}$-surgery on S^3 along the pretzel knot $P(-2,3,7)$. If $10p+1$ is square-free, then M'_p is a hyperbolic 3-manifold that admits no weakly symplectically fillable contact structures.*

Note that there are infinitely many prime integers in form of $10p+1$, due to Dirichlet's theorem on arithmetic progressions. Thus, we have obtained another infinite family of hyperbolic 3-manifold that admit no weakly symplectically fillable contact structures. By the same method, one can show that the 10-surgery on S^3 along the pretzel knot $P(-2,3,7)$ does not admit any weakly symplectically fillable contact structures either.

The pretzel knot $P(-2,3,7)$ has the 4-ball genus $g_s = 5$ and the maximal Thurston-Bennequin invariant $TB = 9$. By [LiSt2, Theorem 1.1], the r-surgery on S^3 along the pretzel knot $P(-2,3,7)$, $P(-2,3,7)_r$, admits a tight contact structure whenever $r > 9$. By Theorem 1.4, all tight contact structures on M'_p, where $10p+1$ is square-free, are not weakly symplectically fillable. Also, Theorem 1.4 provides infinite examples of tight but not fillable contact structures. Admittedly, such examples of contact structures have been constructed on certain toroidal 3-manifolds and some hyperbolic 3-manifolds; see for example [EtH2], [LiSt1] and [BEt]. One should not be confused by the examples in [BEt] with our examples, where it is not known whether their examples admit other fillable structures at all.

Remark 1.5. We compute Heegaard Floer homology with $\mathbb{F} = \mathbb{Z}/2\mathbb{Z}$ coefficients. Thus, by "L-space", we mean "$\mathbb{Z}/2\mathbb{Z}$-L-space". The results of Ozsváth-Szabó on L-spaces in [OzSz3] also apply to $\mathbb{Z}/2\mathbb{Z}$-L-spaces.

Remark 1.6. After writing this paper, we are informed by John Etnyre that Amey Kaloti and Bülent Tosun have independently found surgeries on $P(-2,3,2q+1)$ that admit no fillable contact structures [KT].

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2. Preliminaries

To begin with, we show that M'_p, $p \geq 1$, and M_q, $q \geq 4$, are hyperbolic.

Lemma 2.1. *Suppose $q \geq 4$ and $p \geq 1$ are integers. Then M_q and M'_p are hyperbolic 3-manifolds.*

Proof. Note that the pretzel knot $P(-2,3,2q+1)$ is a hyperbolic knot. Obviously, M_q is neither S^3 nor $S^1 \times S^2$. It is easy to know that the pretzel knot $P(-2,3,2q+1)$ is strongly invertible. So by [Eu, Theorem 4], the cabling conjecture is true for $P(-2,3,2q+1)$. Hence M_q is irreducible. By [Ma, Theorem 1.1], $P(-2,3,2q+1)$ admits no non-trivial cyclic surgery. So M_q is not a lens space. By [Me, Theorem 1.1], the small Seifert fibered surgeries on $P(-2,3,2q+1)$ are of slopes $4q+6$ and $4q+7$. So M_q is not a small Seifert fibered space. By [Wu, Theorem 1.1], the toroidal surgeries on $P(-2,3,2q+1)$ are of slopes $4q+8$. So M_q is atoroidal. Therefore, according to Thurston's geometrization conjecture which was proved by Perelman, [P1, P2, P3], M_q is hyperbolic.

The exceptional surgery coefficients of the pretzel knot $P(-2,3,7)$ are $16, 17, 18, 37/2, 19, 20$ and $1/0$. So M'_p is hyperbolic by the same reason as above. $\square$

According to [OzSz4], the pretzel knot $P(-2,3,2q+1)$ is an L-space knot for $q \geq 1$. Now we show that M'_p, $p \geq 1$, and M_q, $q \geq 4$, are actually L-spaces. To this end, we recall the symmetrized Alexander polynomial of the pretzel knot $P(-2,3,2q+1)$, $q \geq 1$.

Lemma 2.2 (Lemma 2.5, [KL])**.** *The symmetrized Alexander polynomial of the pretzel knot $P(-2,3,2q+1)$ is $(-1)^{q-1} + \sum_{j=1}^{q-1}(-1)^{q-1-j}(t^j + t^{-j}) - (t^{q+1} + t^{-q-1}) + (t^{q+2} + t^{-q-2})$, where $q \geq 1$ is an integer.*

For any rational number r, we denote the r-surgery on K by K_r. According to [OwSt1], when r is an integer, the Spin^c structures on K_r can be indexed by integers i's with $|i| \leq r/2$.

Lemma 2.3. *Suppose $q \geq 4$ and $p \geq 1$ are integers. Then M_q and M'_p are L-spaces.*

Proof. For any L-space knot, by [OzSz3, Theorem 1.2] and [OzSz4, Theorem 1.2], the Seifert genus is the power of the leading term in the symmetrized Alexander polynomial. So by Lemma 2.2, the Seifert genus of $P(-2,3,2q+1)$ is $q+2$, for $q \geq 1$. According to [OzSz6, Proposition 9.6], M'_p, $p \geq 1$, and M_q, $q \geq 4$, are L-spaces. In fact, for any L-space knot K, the surgery manifold K_r with $r \in \mathbb{Q}$ is an L-space for all $r \geq 2g(K) - 1$, where $g(K)$ is the Seifert genus of K. $\square$

We will show that any of M_q with $q \geq 4$ and M'_p with $p \geq 1$ cannot bound negative-definite 4-manifolds. We utilize the following theorem of Owens and Strle.

Theorem 2.4 (Theorem 2, [OwSt1])**.** *Let Y be a rational homology sphere with $|H_1(Y;\mathbb{Z})| = \delta$. If Y bounds a negative-definite 4-manifold X, and if either δ is square-free or there is no torsion in $H_1(X;\mathbb{Z})$, then*

$$\max_{\mathfrak{t} \in \mathrm{Spin}^c(Y)} 4d(Y,\mathfrak{t}) \geq \begin{cases} 1 - 1/\delta & \text{if } \delta \text{ is odd,} \\ 1 & \text{if } \delta \text{ is even.} \end{cases}$$

Suppose K is an L-space knot. Then the d-invariants of integral surgeries on K can be computed by the following formula.

Theorem 2.5 (Therem 6.1, [OwSt1]). *Given $n \in \mathbb{N}$ and $\forall |i| \leq n/2$, we have*

$$\begin{aligned} d(K_n, i) &= d(U_n, i) - 2t_i(K) \\ &= \frac{(n-2|i|)^2}{4n} - \frac{1}{4} - 2t_i(K), \end{aligned}$$

where U is the unknot, the torsion coefficient

$$t_i(K) = \sum_{j>0} j a_{|i|+j},$$

and a_k is the coefficient of t^k in the normalized symmetrized Alexander polynomial $\Delta_K(t)$ of K.

For positive rational surgeries on a knot K, the d-invariants can be computed by the following formula given by Ni and Wu.

Proposition 2.6 (Proposition 1.6, [NW]). *Suppose g, h are positive coprime integers and fix $0 \leq i \leq g-1$. Then,*

$$d(K_{g/h}, i) = d(L(g,h), i) - 2\max\{V_{\lfloor \frac{i}{h} \rfloor}, H_{\lfloor \frac{i-g}{h} \rfloor}\}.$$

Here, V_s and H_s with $s \in \mathbb{Z}$ are knot invariants coming from knot Floer complex; see [NW]. In fact, $H_s = V_{-s}$. Thus, we have the following formula,

$$d(K_{g/h}, i) = d(L(g,h), i) - 2\max\{V_{\lfloor \frac{i}{h} \rfloor}, V_{-\lfloor \frac{i-g}{h} \rfloor}\}. \tag{2.1}$$

3. Integral surgery on $P(-2, 3, 2q+1)$

We compute the torsion coefficients and the d-invariants of M_q as follows.

Lemma 3.1. *For any integer $k \geq 1$, the pretzel knot $K = P(-2, 3, 4k+3)$ has the following formulas for the torsion coefficients $t_j(K)$:*

$$t_j(K) = \begin{cases} k+1-\lfloor \frac{j}{2} \rfloor & j = 0, \ldots, 2k+1, \\ 1 & j = 2k+2, \\ 0 & j \geq 2k+3. \end{cases}$$

For any integer $k \geq 1$, the pretzel knot $K = P(-2, 3, 4k+1)$ has the following formulas for the torsion coefficients $t_j(K)$:

$$t_j(K) = \begin{cases} k+1-\lfloor \frac{j+1}{2} \rfloor & j = 0, \ldots, 2k, \\ 1 & j = 2k+1, \\ 0 & j \geq 2k+2. \end{cases}$$

Proof. Suppose $K = P(-2, 3, 4k+3)$. By definition, we have the recursion relation for $t_j(K)$,

$$t_{j-1}(K) = t_j(K) + \sum_{i \geq j} a_i(K), \quad \forall j \geq 1.$$

By Lemma 2.2, we have that

$$\sum_{i \geq j} a_i(K) = \begin{cases} 1 & \text{if } j = 2k+3, \text{ or } j = 2i \text{ for all } 0 \leq i \leq k, \\ 0 & \text{if } j = 2k+2, \text{ or } j = 2i+1 \text{ for all } 0 \leq i \leq k. \end{cases}$$

It is also easy to see $t_j(K)=0$ for all $j\geq 2k+3$ and $t_{2k+2}(K)=1$. Combing this with the recursion formula, we can prove the result for $K=P(-2,3,4k+3)$.

Similarly, we can prove the results for $K=P(-2,3,4k+1)$. □

Lemma 3.2. *For all integers $q\geq 4$, all the d-invariants on M_q are negative.*

Proof. Suppose $q=2k+1$. By Lemma 3.1 and Theorem 2.5, for all $0\leq j\leq 2k+1$,

$$\begin{aligned}
d(K_{4k+5},\pm j) &= \frac{(4k+5-2j)^2}{4(4k+5)}-\frac{1}{4}-2(k+1-\lfloor\frac{j}{2}\rfloor)\\
&\leq \frac{4k+5}{4}-j+\frac{j^2}{4k+5}-\frac{1}{4}-2(k+1-\frac{j}{2})\\
&= -k-1+\frac{j^2}{4k+5}\\
&\leq -k-1+\frac{(2k+1)^2}{4k+5}\\
&< 0.
\end{aligned}$$

Furthermore,

$$d(K_{4k+5},\pm(2k+2))=\frac{1}{4(4k+5)}-\frac{1}{4}-2<0.$$

Thus, all the d-invariants are negative for the surgery K_{4k+5} when $K=P(-2,3,4k+3)$.

The case when $q=2k$ is similar. □

According to Theorem 2.4 and Lemma 3.2, we have the following lemma.

Lemma 3.3. *Suppose $q\geq 4$ and $2q+3$ is square free, then M_q cannot bound any negative-definite 4-manifold.*

Now we turn back to the proof of the Theorem 1.2.

Proof of Theorem 1.2. We will prove a slightly stronger result that M_q does not admit any weak symplectic semi-fillings. Suppose on the contrary that M_q admits a contact structure which has a weak symplectic semi-filling X. Since M_q is an L-space, by [OzSz3, Theorem 1.4], X must have only one boundary component and have $b_2^+(X)=0$. Since M_q is a rational homology sphere, the intersection form of X is non-degenerate, and thus negative-definite. This contradicts with Lemma 3.3. We finish the proof of Theorem 1.2. □

4. Rational surgeries on $P(-2,3,7)$

The definition of the invariants V_s's from Equation (2.1) comes from the complex A_s^+ that appears in the surgery formulas in Heegaard Floer homology. In the case of L-space knots we can compute them using the Alexander polynomials.

Lemma 4.1. *Suppose K is an L-space knot and its symmetrized Alexander polynomial is $\Delta_K(t)$. We rewrite it in the form*

$$\frac{t}{t-1}\cdot\Delta_K(t)=\sum_i a_i\cdot t^i.$$

Then, we have the following formulas for computing $V_s, \forall s\in\mathbb{Z}$,

$$V_s=\sum_{i\geq s+1}a_i.$$

Proof. Let C denote the knot Floer complex $CFK^{\infty}(K)$ induced by a doubly-pointed Heegaard diagram of K. We have the following commutative diagram where the two rows are short exact sequence of complexes,

$$\begin{tikzcd} 0 \arrow[r] & C\{\max(i,j-s)<0\} \arrow[r] \arrow[d] & C \arrow[r] \arrow[d] & C\{\max(i,j-s)\geq 0\} \arrow[r] \arrow[d] & 0 \\ 0 \arrow[r] & C\{i<0\} \arrow[r] & C \arrow[r] & C\{i\geq 0\} \arrow[r] & 0. \end{tikzcd}$$

Since $A_s^- = C\{\max(i,j-s)\leq 0\} \cong C\{\max(i,j-s)<0\}$ and $A_{+\infty}^-$ is isomorphic to $C\{i\leq 0\}\cong C\{i<0\}$, the above diagram translates into

$$\begin{tikzcd} 0 \arrow[r] & A_s^- \arrow[r] \arrow[d, "I_s"'] & C \arrow[r] \arrow[d] & A_s^+ \arrow[r] \arrow[d, "v_s"] & 0 \\ 0 \arrow[r] & A_{+\infty}^- \arrow[r] & C \arrow[r] & B_s^+ \arrow[r] & 0. \end{tikzcd}$$

Since K is an L-space knot, we have that $A_s^- = \{\mathbf{x}\in CFK^{\infty}(K)|A(\mathbf{x})\leq s\}$ has homology $\mathbb{F}[U]$. By passing to the long exact sequence of homologies, one can show that the induced map on homology $(I_s)_*:\mathbb{F}[U]\to\mathbb{F}[U]$ is a multiplication of some power U^{n_s} and $V_s = n_s$.

From the long exact sequence induced by the short exact sequence $0\to A_{i-1}^- \xrightarrow{\iota_{i-1}} A_i^- \to A_i^-/A_{i-1}^- \to 0$, we have the following exact sequence $0\to \mathrm{coker}((\iota_{i-1})_*)\to H_*(A_i^-/A_{i-1}^-)\to \ker((\iota_{i-1})_*)\to 0$. From $I_{i-1} = I_i\circ\iota_{i-1}$, it follows that ι_{i-1} cannot be 0 acting on homology. Thus, $\ker((\iota_{i-1})_*) = 0$, and $H_*(A_i^-/A_{i-1}^-) = \mathrm{coker}((\iota_{i-1})_*)$. Hence, we have $n_{i-1}-n_i = \chi(A_i^-/A_{i-1}^-)$.

Since $CFK^-(K,i) = A_i^-/A_{i-1}^-$, by Proposition 9.2 in [OzSz5], we have that

$$\begin{aligned} n_s &= \sum_{i\geq s+1}\chi(HFK^-(K,i)) \\ &= \sum_{i\geq s+1} a_i. \end{aligned}$$

□

Lemma 4.2. *Let K be the $P(-2,3,7)$ pretzel knot. For all $p\geq 1$, all the d-invariants on $K_{\frac{10p+1}{p}}$ are non-positive.*

Proof. By Equation (2.1), we have $\forall 0\leq i\leq 10p$,

$$d(K_{\frac{10p+1}{p}}, i) = d(L(10p+1,p),i) - 2\max\{V_{\lfloor\frac{i}{p}\rfloor}, V_{-\lfloor\frac{i-10p-1}{p}\rfloor}\}.$$

By Lemma 4.1 and Lemma 2.2, we know

$$V_0 = V_1 = 2,\quad V_2 = V_3 = V_4 = 1,\quad V_p = 0,\ \forall p\geq 5.$$

By Proposition 4.8 in [OzSz1], we have

$$\begin{aligned} d(L(10p+1,p),i) &= -\frac{1}{4} + \frac{(11p-2i)^2}{4p(10p+1)} - d(L(p,1),j) \\ &= \frac{(11p-2i)^2}{4p(10p+1)} - \frac{(p-2j)^2}{4p}, \end{aligned}$$

with $0\leq j\leq p-1$ being the residue of i modulo p.

Now we will discuss the correction terms case by case.

(1) If $0 \leq i \leq 2p-1$, then $\lfloor \frac{i}{p} \rfloor = 1$. Then, $V_{\lfloor \frac{i}{p} \rfloor} = 2$. Thus,

$$\max\{V_{\lfloor \frac{i}{p} \rfloor}, V_{-\lfloor \frac{i-10p-1}{p} \rfloor}\} = 2.$$

Therefore,

$$d(K_{\frac{10p+1}{p}}, i) \leq \frac{(11p)^2}{4p \cdot 10p} - 2 \cdot 2 < 0.$$

(2) If $2p \leq i \leq 5p-1$, then $\lfloor \frac{i}{p} \rfloor = 2, 3, 4$. Then, $V_{\lfloor \frac{i}{p} \rfloor} = 1$. So

$$\max\{V_{\lfloor \frac{i}{p} \rfloor}, V_{-\lfloor \frac{i-10p-1}{p} \rfloor}\} = 1.$$

In addition, since $2p \leq i \leq 5p-1$, we have $(11p-2i)^2 \leq (7p)^2$. Thus,

$$d(K_{\frac{10p+1}{p}}, i) \leq \frac{(7p)^2}{4p \cdot 10p} - 2 \cdot 1 < 0.$$

(3) If $5p \leq i \leq 6p-1$, then let

$$i = 5p + j,$$

where $0 \leq j \leq p-1$.

Thus,

$$\begin{aligned} d(K_{\frac{10p+1}{p}}, i) &\leq \frac{(11p-2i)^2}{4p \cdot (10p+1)} - \frac{(p-2j)^2}{4p} \\ &= \frac{(p-2j)^2}{4p \cdot (10p+1)} - \frac{(p-2j)^2}{4p} \\ &\leq 0. \end{aligned}$$

(4) If $i = 6p$, then

$$d(K_{\frac{10p+1}{p}}, i) \leq \frac{p^2}{4p \cdot (10p+1)} - \frac{p^2}{4p} < 0.$$

(5) If $6p+1 \leq i \leq 8p+1$, then $-\lfloor \frac{i-10p-1}{p} \rfloor = 2, 3, 4$. Then, $V_{-\lfloor \frac{i-10p-1}{p} \rfloor} = 1$. So

$$\max\{V_{\lfloor \frac{i}{p} \rfloor}, V_{-\lfloor \frac{i-10p-1}{p} \rfloor}\} = 1.$$

In addition, since $6p+1 \leq i \leq 8p+1$, we have $(11p-2i)^2 \leq (7p)^2$. Thus,

$$d(K_{\frac{10p+1}{p}}, i) \leq \frac{(7p)^2}{4p \cdot 10p} - 2 \cdot 1 < 0.$$

(6) If $8p+2 \leq i \leq 10p$, then either $-\lfloor \frac{i-10p-1}{p} \rfloor = 1$. Then, $V_{-\lfloor \frac{i-10p-1}{p} \rfloor} = 2$. Thus,

$$\max\{V_{\lfloor \frac{i}{p} \rfloor}, V_{-\lfloor \frac{i-10p-1}{p} \rfloor}\} = 2.$$

Therefore,

$$d(K_{\frac{10p+1}{p}}, i) \leq \frac{(11p)^2}{4p \cdot 10p} - 2 \cdot 2 < 0.$$

□

Now we are ready to prove Theorem 1.4.

Proof of Theorem 1.4. We follow the same line as in Theorem 1.2. By Lemma 2.1, we have M'_p are all L-spaces. As long as $10p+1$ is square-free, by Theorem 2.4, M'_p does not bound any negative definite 4-manifolds and thus does not admit any weakly symplectically fillable contact structures. We finish the proof. □

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